Condensate fraction of molecules for a spin mixture of ultracold fermionic atoms

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The condensate fraction of molecules for ultracold Fermi gases is investigated for the magnetic field below the Feshbach resonant magnetic field. Assuming that there is no loss of particles and energy during the adiabatic magnetic-field sweep, a simple theory is used to interpret the measured condensate fraction in the experiments by JILA group (Phys. Rev. Lett. 92, 040403 (2004)) and MIT group (Phys. Rev. Lett. 92, 120403 (2004)). Our theory shows that the condensate fraction of molecules is dependent on the initial condition of the system and especially on the process of the magnetic-field sweep.

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The evidence for Bose-Einstein condensates of diatomic molecules has been finally given in several remarkable experiments [1, 2, 3] which will obviously lead to intensive theoretical and experimental researches on the ultracold Fermi gases. The magnetic-field Feshbach resonance [4, 5] plays an important role in most of the recent experiments on ultracold Fermi gases because it can change both the strength and sign of the scattering length a between fermionic atoms with different internal freedom. On the side of strongly repulsive interaction (BEC side), there is molecule which is short-range fermionic pairs. On the side of strongly attractive interaction (BCS side), one expects that there are fermionic pairs analogously to the electronic Cooper pairs in superconductor. The Feshbach resonance has given us an important opportunity to investigate the pairing phenomena in the ultracold Fermi gases especially the BCS-BEC crossover which has been discussed in a lot of theoretical researches [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Recently, the BCS-BEC crossover is investigated by several experiments [18, 19, 20, 21, 22, 23, 24] such as the superfluid hydrodynamics [22, 23] and pairing gap [24].

The condensate fraction near a Feshbach resonance has been investigated experimentally in [18, 19] by adiabatically sweeping the magnetic field to different value, and the condensate fraction is then measured by a following fast decreasing of the magnetic-field to convert the dimers of fermionic atom pairs into bound molecules. Above the Feshbach resonant magnetic field, the condensate fraction observed by JILA group [18] is investigated in [15] based on the model of a molecular BEC. On the Feshbach resonance, a simple theoretical model of a mixture of Fermi degenerate gas and dimeric gas [17] is proposed to explain the high condensate fraction observed in [19]. Below the Feshbach resonant magnetic field, the condensate fraction of molecules is investigated experimentally by JILA group [18] for ^{40}K and MIT group [19] for ^{6}Li with different magnetic-field sweep process. As far as we know, below the Feshbach resonant magnetic field, presently there is no theoretical interpretation for the condensate fraction of molecules measured in [18, 19]. In the experimental work of JILA group [18], the condensed atom pairs was found to be at most 15%, while the MIT group [19] observed a much higher fraction of condensed atom pairs below the Feshbach resonant magnetic field. The high condensate fraction observed by MIT group has been confirmed by other group such as the experiment by Bartenstein et al [20] for ^{6}Li. Thus, there is an urgent need for a theoretical model to interpret this great difference for different groups. In the present work, omitting the loss of particles and thus the loss of energy during the adiabatic magnetic-field sweep, a very simple theoretical model is used to interpret the experimental results in both [18] for ^{40}K and [19] for ^{6}Li. It is found that the condensate fraction of molecules is dependent on the initial condition of the system and especially on the process of the magnetic-field sweep.

The magnetic-field Feshbach resonance [4, 5] can change both the strength and sign of the atomic interaction. Near the Feshbach resonance, the scattering length is $a(B) = a_{bg}[1 - w/(B - B_0)]$ with a_{bg} being the background scattering length and B denoting the magnetic field with Feshbach resonant magnetic field B_0. Below the Feshbach resonance, the energy of the molecular state can be estimated well to be $-\hbar^2/ma^2$ [25] with m the mass of the fermionic atom.

During an evaporative cooling process, it is well-known that there is a significant loss of energy and particles. For most of the recent experiments, however, the BCS-BEC crossover is investigated for the ultracold gases prepared at an initial magnetic field after different evaporative cooling process. After the evaporative cooling and during the following adiabatic magnetic-field sweep, the loss of particles can be omitted and thus we omit the loss of the energy of the system. In the present experiments on the BCS-BEC crossover, the temperature is much smaller than the Fermi temperature. Thus, in this work, we investigate the ultracold gases at zero temperature. At zero temperature and during an adiabatic magnetic-field sweep process, omitting the loss of particles and thus the loss of energy, in the present work a simple theory is used to investigate the condensate fraction during the adiabatic magnetic-field sweep.

Assume that N denotes the total number of fermionic

atoms in the absence of molecules and N_m is the total number of molecules. Assume further that $N_\uparrow$ and $N_\downarrow$ are the number of fermionic atoms with different internal freedom. In the present experimental works on the BCS-BEC crossover, an equal mixture of fermionic atoms with different internal freedom is prepared in an optical trap. In this case, one has the following confinement condition:

$$N = 2N_\uparrow + N_m. \tag{1}$$

To use the assumption that the loss of energy can be omitted, we calculate the overall energy of the system below and on the Feshbach resonant magnetic field. Below the Feshbach resonant magnetic field with $|a|/\overline{l} < 1$ ($\overline{l}$ is the average distance between atoms), the chemical potential of the Fermi gas is given by

$$\mu_\uparrow(N_\uparrow, B) = \frac{\left(6\pi^2\right)^{2/3}\hbar^2}{2m} n_\uparrow^{2/3} + \frac{1}{2} g_F n_\uparrow + V_F, \tag{2}$$

where the coupling constant $g_F = 4\pi\hbar^2 a/m$, and $n_\uparrow$ is the density distribution of the fermionic atoms in an internal freedom. $V_F = m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)/2$ is the external potential of the fermionic atom. The overall energy $E_F(N_\uparrow, N_\downarrow, B)$ of the interacting Fermi gases is given by

$$E_F(N_\uparrow, N_\downarrow, B) = \frac{(6N_\uparrow)^{4/3}\hbar\omega_{ho}}{4} + g_F \int n_\uparrow^2 dV, \tag{3}$$

where $\omega_{ho} = (\omega_z\omega_y\omega_z)^{1/3}$ is the geometric average of the harmonic frequencies.

At zero temperature, by using the Thomas-Fermi approximation [31], the overall energy of N_m molecules in the condensate is given by

$$E_m(N_m, B) = N_m\varepsilon_m + \frac{5N_m\mu_m^0}{7}, \tag{4}$$

where the energy of the molecular state is $\varepsilon_m = -\hbar^2/ma^2$. In the above expression, $\mu_m^0 = \hbar\omega_{ho}\left(15N_m a_m/a_{ho}^m\right)^{2/5}/2$ with the molecular scattering length $a_m \approx 0.6a$ [32] and the harmonic oscillator length $a_{ho}^m = \sqrt{\hbar/2m\omega_{ho}}$. The overall energy of the mixture gases of fermionic atoms and molecules is then

$$E_{F-m} = E_F(N_\uparrow, N_\downarrow, B) + E_m(N_m, B). \tag{5}$$

On resonance, the absolute value of the scattering length a is divergent and thus much larger than the average distance $\overline{l}$ between particles. In this situation, the ultracold gas can be described in the unitarity limit [26, 27, 28, 29] where there is a universal behavior for the system. By using the local density approximation, the general form of the chemical potential for the Fermi gas in the unitarity limit can be given by the dimensionality analysis:

$$\mu_\uparrow^{res} = (1+\beta)\frac{\left(6\pi^2\right)^{2/3}\hbar^2}{2m} n_\uparrow^{2/3} + V_F, \tag{6}$$

where β is the correction due to the strong interaction between fermionic atoms with different internal freedom on resonance. The parameter β has been investigated by both experiments [20, 21, 30] and theories [26, 27]. On resonance, a quantum Monte Carlo calculation [27] gives $\beta = -0.56$. From Eq. (6), the overall energy of the system on resonance is given by

$$E_{res}(N) = (1+\beta)^{1/2}\frac{(3N)^{4/3}}{4}\hbar\omega_{ho}. \tag{7}$$

We turn to discuss the condensate fraction during the adiabatic magnetic-field sweep with different initial condition of the system. Firstly we consider the case that the initial system in thermal equilibrium is prepared on resonance, and then the magnetic field is lowered adiabatically below the Feshbach resonant magnetic field B_0. In this situation, if the loss of the energy during the magnetic-field sweep is omitted, one has the following confinement condition on the condensate fraction

$$E_F(N_\uparrow, N_\downarrow, B) + E_m(N_m, B) = E_{res}(N). \tag{8}$$

Combining with Eq. (1), one can get the condensate fraction of molecules $x = 2N_m/N$ at zero temperature for the magnetic field below B_0.

In the experiment by JILA group [18] for ^{40}K, the scattering length is $a = 174a_0(1 - 7.8/(B - B_0))$ with a_0 being the Bohr radius and $B_0 = 202.1$G being the Feshbach resonant magnetic field. In addition, according to the experiment in [18], $N \approx 2\times10^5$ and $\omega_x/2\pi = \omega_y/2\pi = 320$ Hz, $\omega_z = \omega_x/79$. In [18], the system was initially prepared at a magnetic field 235.6 G far above the Feshbach resonance. The magnetic field was then swept across B_0 adiabatically and especially there was a holding time of about 3 ms on resonance (see the inset of figure 2 in [18]). The adiabaticity of this sweep process can be verified by estimating the relaxation time $\gamma \approx 1/\sqrt{2}n_\uparrow\sigma\overline{v}$ with the total cross section $\sigma = 8\pi a^2$ and the mean value of the velocity of the thermal atoms $\overline{v} = \sqrt{8k_BT/\pi m}$. A simple calculation shows that for the regime of the magnetic field investigated in [18], the relaxation time is smaller than 0.1 ms, while the sweep and holding time in [18] is larger than 1 ms. One should note that the condensate fraction was measured in [18] by lowering quickly (non-adiabatically) the magnetic field which results in a measured loss of 50% of the molecules [18].

In [18], there was a holding time of about 3 ms on resonance. On the other hand, the divergent scattering length on resonance means that the relaxation time is extremely small and a small loss of particles on resonance

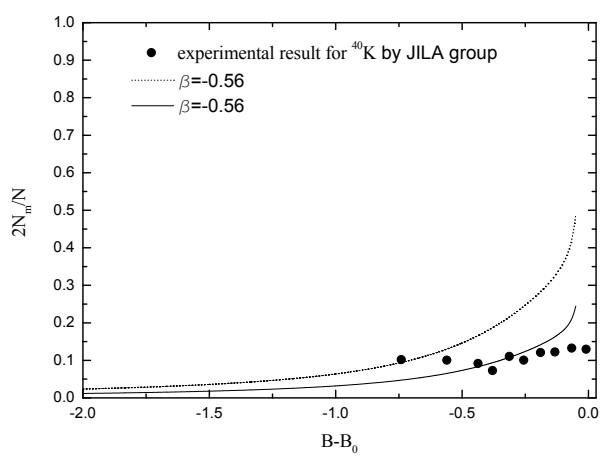

FIG. 1: By using the experimental parameters of JILA group [18], shown is the condensate fraction $x = 2N_m/N$ for $^{40}K_2$ molecules at zero temperature and below the Feshbach resonant magnetic field B_0. The parameter $\beta = -0.56$ is the correction to the energy of the ultracold gases on resonance. The dotted line is the condensate fraction obtained from Eq. (8), while the solid line is the condensate fraction obtained by considering the loss of molecules during the probing procedure. The solid circles are the measured condensate fraction by JILA group.

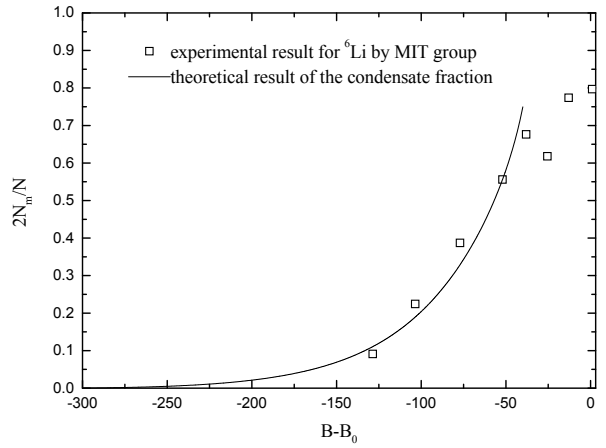

FIG. 2: The solid line shows the calculated condensate fraction of molecules at zero temperature for an experimental process of MIT group [19] where the ultracold gases with condensate fraction 0.56 were initially prepared at the magnetic field 770 G below the Feshbach resonant magnetic field B_0. We see that the solid line based on the energy conservation given by Eq. (9) agrees well with the experimental result shown by □ with a hold time of 100 ms.

will change the overall energy of the system effectively. Thus the energy of the ultracold gases on resonance can be determined by $E_{res}(N)$. In this situation, Eqs. (1) and (8) can be used to calculate the condensate fraction x for different magnetic field B ($< B_0$). The dotted line in figure 1 shows the condensate fraction calculated from Eq. (8) by using the above experimental data. We see that at least qualitatively the condensate fraction based on $\beta = -0.56$ agrees with the experimental result shown by the solid circle. The solid line shows the condensate fraction when 50% loss of the molecules is considered during the probing procedure (i.e. lowering the magnetic field quickly) pointed out in [18].

We now consider another case that the system is prepared initially below the Feshbach resonant magnetic field (i.e. on the side of molecular BEC). Assuming that x_{ini} is the condensate fraction at an initial magnetic field B_{ini}, the condensate fraction after lowering adiabatically the magnetic field to a value B is then determined by the following confinement condition when the loss of energy is omitted:

$$E_F\left(N_\uparrow = N_\downarrow = N\left(1-x_{ini}\right)/2, B_{ini}\right) + E_m\left(x_{ini}N/2, B_{ini}\right)$$

$$= E_F\left(N_\uparrow = N_\downarrow = N\left(1-x\right)/2, B\right) + E_m\left(Nx/2, B\right). \quad (9)$$

Different from the experiment by JILA group [18], the experimental results illustrated by □ in figure 3 of the experimental paper by MIT group [19] were observed for the system initially prepared at the magnetic field $B_{ini} = 770$ G [19] which is smaller than B_0. From the experimental data, the condensate fraction at $B_{ini} = 770$ G is $x_{ini} = 0.56$. The condensate fraction of molecules below B_0 was then investigated experimentally by lowering adiabatically the magnetic field. One can easily verify the adiabaticity of the magnetic-field sweep in [19] through the estimation of the relaxation time. The experimental parameters in [19] are $a = -1018a_0\left(1 + 325/\left(B - B_0\right)\right)$ [33] with $B_0 = 822$ G and $N = 2\times10^6$, $\omega_x/2\pi = \omega_y/2\pi = 115 \times \sqrt{25}$ Hz, $\omega_z/2\pi = 1.1 \times \sqrt{25 + 120 \times B/1000}$ Hz.

The solid line in figure 2 shows the theoretical result based on the above equation (9). We see that our theory agrees well with the experimental result of [19] illustrated by □ in figure 2. Different from the analyses for the experiment by JILA group, we do not consider the loss of molecules during the probing procedure because in the experiment by MIT group for ^6Li, the molecules is much more stable comparing with the JILA group with ^{40}K. We believe that both the experimental data of JILA group [18] and MIT group [19] are reliable. The large difference of the condensate fraction in [18] and [19] is due to different experimental process.

In the above theoretical investigation for the experiment by MIT group, the harmonic frequency ω_z is dependent on the magnetic field. In fact, due to the B-dependent harmonic frequency ω_z, there is an effective work on the system by the magnetic field due to the continuous changing of the ground state of the system. This effective work can be omitted because $\Delta\omega_z/\omega_z < 4\%$

between the regime of magnetic field 770 G and 700 G investigated in the experiment. In figure 2, the agreement between theoretical result and experimental data shows clearly that the one can safely omit the effective work done by the magnetic field.

For the condensate fraction discussed here, the term $N_m\varepsilon_m$ in the overall energy of the molecular gas $E_m(N_m, B)$ plays a special role because it is negative and dependent strongly on the magnetic field. Below the Feshbach resonance and with the decreasing of the magnetic field, the term ε_m decreases significantly. When the loss of energy is omitted, to maintain the energy conservation, there should be a decreasing of the number of molecules when lowering adiabatically the magnetic field, which has been shown clearly in [18] and [19]. One should note that the overall energy of the ultracold gases is investigated here for the case of weakly interacting gases with $a/\bar{l} < 1$ and the case of the divergent scattering length. Thus the theoretical result for the condensate fraction obtained here does not hold in the strongly interacting gases with $a/\bar{l} > 1$, and this is the reason why we do not give the condensate fraction for $a/\bar{l} > 1$.

In summary, the simple theory developed here is used to interpret different experiments by JILA group [18] and MIT group [19] on the condensate fraction. Obviously, the condensate fraction discussed here is dependent on the magnetic-field sweep process and initial condition of an experiment. When the ultracold gases are prepared at the magnetic field below B_0, after different evaporative cooling one can get relatively arbitrary condensate fraction. One can check our theory further by investigating the condensate fraction in experiments for different magnetic-field sweep process and different initial condition of the system. It will be an interesting future work to investigate the condensate fraction of the fermionic atom pairs above the Feshbach resonant magnetic field based on the assumption that the loss of energy can be omitted.

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